N71-30850

NASA TECHNICAL TRANSLATION NASA TT F-13,747

PLASMA WAVES AND SUPERCONDUCTIVITY IN QUANTIZING SEMICONDUCTOR (SEMIMETAL) FILMS AND LAYERED STRUCTURES

E. A. Pashitskiy, Yu. A. Romanov

Translation of "Plazmennyye volny i sverkhprovodimost' v kvantuyushchikh poluprovodnikovykh (polumetallicheskikh) plenkakh i sloistykh strukturakh," Ukrainskiy Fizicheskiy Zhurnal, Vol. 15, No. 10, 1970, pp. 1594-1606

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D.C. 20546 JULY 1971

PLASMA WAVES AND SUPERCONDUCTIVITY IN QUANTIZING SEMICONDUCTOR (SEMIMETAL) FILMS AND LAYERED STRUCTURES

E. A. Pashitskiy, Yu. A. Romanov

ABSTRACT. It is shown that a specific mechanism of coupling degenerate conductance electrons, the result of virtual acoustic plasmon exchange, occurs in quantizing semiconductor (semimetal) films and layered structures with a reasonably high mobility of free carriers. As a result, the critical temperature of the superconductive transition in such systems can be noticeably higher than in massive specimens.

Introduction /1594*

Today, thanks to the progress that has been made in microelectronics, as well as in a number of applied fields of experimental physics, a great deal of attention is being devoted to research on the physical properties of superthin metal and semiconductor films, and in particular to the study of the size effects of quantization in films (see the summaries in [1, 2]).

Of particular interest is the phenomenon of a rise in the critical temperature of a superconducting junction, T_c, in thin films [3-7] and in multilayered structures consisting of alternating metal and dielectric layers [8], as well as the occurrence of superconductivity in the films of certain semimetals (bismuth, for example). The cause of this phenomenon can be the natural intensification of phonon superconductivity attributable to the existence of a new crystallographic phase in the film state [6], or to the strengthening of the role of the surface (Rayleigh) phonons localized near the film boundaries, and in flaws [7], as well as to the manifestation of various non-phonon mechanisms of superconductivity associated, in particular, with the polarization excitation of atoms (that is, with the virtual excitons) in dielectric coatings (oxides) on the surface of the film [5, 9-11].

Reference [12] contains the suggestion that it is possible for T_c to rise in layered semiconductor structures (of the "sandwich" type) because of the interaction between the conductance electrons in a thin (quantizing) film,

*Numbers in the margin indicate pagination in the foreign text.

and the surface plasma oscillations relative to "heavy" carriers (holes) at the interfaces of the layers-heterojunctions.[1] Reference [15] proposed the use of granulated (dispersed) metal films as the active coating for the "sandwich" in order to increase the two-dimensional superconductivity. These films are characterized by abnormally high polarizability, and by the broad spectrum of the collective dipole oscillations of the electrons in the granules [16].

On the other hand, as reference [17] has pointed out, the quantization of the transverse motion of the electrons in semiconductor and semimetal films, when carrier mobility is high enough, results in the appearance of slowly decaying branches of quasi-neutral plasma oscillations with an acoustic dispersion law, attributable to the presence of several groups of electrons with different limit (Fermi) velocities. This is why, according to reference [18], an additional (with respect to the phonon) mechanism for pairing electrons as /1595
a result of the virtual acoustic plasmon exchange can appear in quantizing films.[2]

However, references [15, 18] did not take the delay in the "electron–plasmon" interaction into consideration because a review had been made in an adiabatic approximation by the Bardeen, Cooper, Schrieffer-Bogolyubov method [19, 20]. (Moreover, reference [18] did not take the Coulomb repulsion of electrons into consideration).

This paper utilizes the more sequential method of Green's temperature functions [21, 22] because on the one hand doing so makes it possible to give proper consideration to the collective (dynamic) effects of delay and Coulomb interaction, and, on the other, to obtain directly an explicit (albeit approximate) expression for the critical temperature of a superconducting junction in quasi-two-dimensional layered structures.

---

1. We should note that [13] also pointed out the positive role of the surface plasmons in the formation of bound electron pairs, and that collective surface oscillations of free carriers in a p-n junction were reviewed in [14].

2. Reference [18] also considered acoustic plasma waves in quantizing filaments and discussed the possibility of intensifying the one-dimensional superconductivity in such systems.

1. Acoustic and surface plasma waves.

Let us consider the collective oscillations of a degenerated electron plasma[3] in a thin (quantizing) semiconductor, or semimetal film, of thickness d, bounded on both sides by a homogeneous medium with permittivity $\varepsilon_0(q,\omega)$. The dispersion equation for the oscillations in such system is in the form [17, 23]

$$\frac{2}{d}\sum_{\nu}\frac{q}{(q^2+q_\nu^2)\,\varepsilon(\mathrm{q},q_\nu,\omega)}=-\frac{1}{\varepsilon_0(\mathrm{q},\omega)}, \tag{1.1}$$

where

$$\varepsilon(\mathrm{q},q_\nu,\omega)=\varepsilon_l+\frac{8\pi e^2}{(q^2+q_\nu^2)\,d}\sum_n\int\frac{d^2k}{(2\pi)^2}\times$$
$$\times\frac{f(\mathcal{E}_{n+\nu}(\mathrm{k}+\mathrm{q}))-f(\mathcal{E}_n(\mathrm{k}))}{\hbar\omega+\mathcal{E}_n(\mathrm{k})-\mathcal{E}_{n+\nu}(\mathrm{k}+\mathrm{q})+i0}, \tag{1.2}$$
$$\mathcal{E}_n(\mathrm{k})=\frac{\hbar^2k^2}{2m_\parallel}+\frac{\hbar^2}{2m_\perp}\left(\frac{n\pi}{d}\right)^2,$$

in which

$m_\parallel(m_\perp)$ is the longitudinal (transverse) effective mass of the conductance electron;

$n = \pm 1, \pm 2, \ldots$, $f(\mathcal{E})$ is a Fermi function;

$q_\nu = q_\nu^{(c)} \equiv 2\nu\pi/d$ for symmetrical (tangential) oscillations;

$q_\nu = q_\nu^{(a)} \equiv (2\nu + 1)\pi/d$ for antisymmetrical (normal) oscillations;

$\nu = 0, \pm 1, \pm 2, \ldots$, ε_i is the permittivity of the crystal lattice ($\varepsilon_i \approx$ const.)

We should point out that when the spatial dispersion in the transverse direction (with respect to the plane of the film) is ignored, Eq. (1.1) will be in the following forms when ε does not depend on q_ν

$$\varepsilon(\mathrm{q},\omega)\,\mathrm{th}\,\frac{qd}{2}+\varepsilon_0(\mathrm{q},\omega)=0 \tag{1.1a}$$

in the case of symmetrical oscillations and /1596

$$\varepsilon(\mathrm{q},\omega)\,\mathrm{cth}\,\frac{qd}{2}+\varepsilon_0(\mathrm{q},\omega)=0 \tag{1.1b}$$

3. This can just as well be the hole plasma of p-type semiconductors (semimetals).

in the case of antisymmetrical oscillations (compare with [12]). Henceforth, for simplicity's sake, we shall limit ourselves to consideration of symmetrical oscillations (see section 2, below).

If we consider the conditions

$$qd \ll 1,\ \hbar\omega \ll \mathcal{E}_{n+1}(\mathrm{k}) - \mathcal{E}_n(\mathrm{k}) \tag{1.3}$$

as having been satisfied, and if we drop all summands in Eq. (1.1) with $v \neq 0$, we will obtain the following dispersion equation for these oscillations[4]

$$\varepsilon(\mathrm{q}, 0, \omega) = -\frac{2\varepsilon_0(\mathrm{q}, \omega)}{qd}, \tag{1.4}$$

where

$$\varepsilon(q, 0, \omega) = \varepsilon_l + \frac{4}{q^2 a_{\parallel} d}\Bigg\{N - $$

$$- \sum_{n=1}^{N} \frac{k_n}{q}\left[\sqrt{\left(\frac{\omega}{qv_n} + \frac{q}{2k_n}\right)^2 - 1} - \sqrt{\left(\frac{\omega}{qv_n} - \frac{q}{2k_n}\right)^2 - 1}\right]\Bigg\}. \tag{1.5}$$

Here N is the number of filled two-dimensional subzones (in the momentum space), found for specified concentrations of electrons, n_o, and film thickness, d, using the relationship

$$\frac{\pi}{12} N(N-1)(4N+1) < n_0 d^3 < \frac{\pi}{12} N(N+1)(4N+5), \tag{1.6}$$

where

$a_{\parallel} = \hbar^2/m_{\parallel}e^2$ is the effective (longitudinal) Bohr radius of the conductance electron;

$k_n = \sqrt{k_F^2 - (n\pi/d)^2}$ is the boundary wave vector;

$v_n = \hbar k_n/m_{\parallel}$ is the cutoff velocity for the electrons in the n^{th} subzone.

In this case, the Fermi vector, k_F, can be found through the expression

$$k_F^2 = \frac{2\pi n_0 d}{N} + \left(\frac{\pi}{d}\right)^2 \frac{(N+1)(2N+1)}{6}. \tag{1.7}$$

---

4. The "Umklapp" processes, that is, the virtual electron transitions between different subzones, were not considered (see [17]).

a. Acoustic plasma waves. Let us consider the permittivity of an external medium with a constant (ε_0 = const).

And let us assume as well that only two subzones (N = 2) in the film are filled. For example, this condition will be satisfied in the case of a film of thickness $d = 2 \cdot 10^{-6}$ cm when the electron concentration in the interval is $5.9 \cdot 10^{17}$ cm^{-3} < n_0 < $2.55 \cdot 10^{18}$ cm^{-3}.

If, in addition to the conditions set forth in Eq. (1.3), the inequality[5] /1597

$$\left|\frac{s^2}{v_1^2} - 1\right| \gg \frac{q}{k_1}\left(\frac{s}{v_1} + \frac{q}{4k_1}\right), \qquad qa_{\parallel} \ll \frac{4}{\varepsilon_0}, \quad q^2 a_{\parallel} d \ll \frac{8}{\varepsilon_i}, \tag{1.8}$$

in which $s = \omega/q$, is satisfied, Eq. (1.4) will reduce to the condition that $\varepsilon(q,0,\omega) = 0$, and, in accordance with Eq. (1.5), will take the form

$$2 - \frac{k_2}{q}\left[\sqrt{\left(\frac{s}{v_2} + \frac{q}{2k_2}\right)^2 - 1} - \sqrt{\left(\frac{s}{v_2} - \frac{q}{2k_2}\right)^2 - 1}\right] + \frac{i}{\sqrt{\frac{v_1^2}{s^2} - 1}} = 0. \tag{1.9}$$

Taking $v_1^2 \gg s^2$, for reasons of simplicity, we obtain the following expression for finding the frequency of the oscillations, ω_q

$$\omega_q^2 \approx q^2 v_2^2 \left[\frac{4}{3} + \frac{q^2}{k_2^2} - i\frac{\omega_q}{qv_1}\left(\frac{4}{9} - \frac{q^2}{k_2^2}\right)\right], \qquad q < q_{lim} \equiv \frac{2}{3}k_2. \tag{1.10}$$

When $q = q_{lim}$, the spectrum of the oscillations merges with the continuous spectrum of single-particle electron excitations. The corresponding limit frequency is equal to

$$\omega_{lim} = \frac{4}{3} q_{lim} v_2 = \frac{8}{9} k_2 v_2. \tag{1.11}$$

---

5. We should point out that the second inequality in Eq. (1.8) permits us to ignore surface waves on film boundaries. On the other hand, the last inequality provides quasineutrality of the volume plasma oscillations, and, when $\varepsilon_i \gg 1$, can prove to be stronger than the condition $qd \ll 1$ in Eq. (1.3).

When $q \gg q_{lim}$ the oscillations can be characterized by an acoustic dispersion law with phase velocity $s = 2v_2/\sqrt{3}$.

It should be pointed out that the oscillations considered for the most part are associated with disturbances of electron density in the second subzone with a lower cutoff velocity ($v_2 < v_i$), and that the role of the electrons in the first subzone reduces to compensating for the space charge, and to resonance absorption of the oscillations.[6]

b. Surface waves. Let us consider the dispersion of the external medium in terms of time and space. If the medium is a metal (semimetal), or is a degenerated semiconductor with a sufficiently high concentration, and with high mobility of the free carriers, the permittivity, $\varepsilon_0(q,\omega)$, can be given in the form

$$\varepsilon_0(q,\omega) \approx \begin{cases} \varepsilon_0\left(1-\frac{\Omega_0^2}{\omega^2}\right), & \omega \gg qv_0, \quad (1.12a) \\ \varepsilon_0\left(1+\frac{\varkappa_0^2}{q^2}\right), & \omega \ll qv_0. \quad (1.12b) \end{cases}$$

where /1598

$\Omega_0 = (4\pi e^2 N_0/\varepsilon_0 m_0)^{1/2}$ is the plasma (Langmuir) frequency;

v_0 is the Fermi velocity;

$\varkappa_0^{-1} = (\varepsilon_0 \mathcal{E}_0/6\pi e^2 N_0)^{1/2}$ is the effective radius of the shielding;

$\mathcal{E}_0$ is the Fermi energy;

N_0 is the concentration;

m_0 is the effective mass of the carrier.

However, in accordance with Eq. (1.5), we have the following for a quantizing film

$$\varepsilon(q,0,\omega) \approx \begin{cases} \varepsilon_l\left(1+\frac{\varkappa_l^2}{q^2}\right), & \omega \ll qv_n, \quad (1.13a) \\ \varepsilon_l\left(1-\frac{\Omega_l^2}{q^2}\right), & \omega \gg qv_n, \quad (1.13b) \end{cases}$$

---

6. Here it is assumed that the damping of the oscillations associated with the finite mobility of the electrons is slight ($\omega_{lim}\tau \gg 1$).

where

$$\varkappa_i^2 = \frac{4N}{\varepsilon_i a_\parallel d}, \qquad \Omega_i^2 = \frac{2}{\varepsilon_i a_\parallel d} \sum_{n=1}^{N} v_n^2 \tag{1.14}$$

(and in particular, when N = 1, the frequency $\Omega_i = \sqrt{4\pi e^2 n_o/\varepsilon_i m_{||}}$).

Substituting Eqs. (1.12) and (1.13) in Eq. (1.4), the equation of dispersion, we obtain the following expression for the frequency of the surface plasma waves in a three-layered "sandwich" type system

$$\Omega_q \approx \begin{cases} \Omega_0\left[1+\frac{\varepsilon_i}{\varepsilon_0}\frac{qd}{2}\left(1+\frac{\varkappa_i^2}{q^2}\right)\right]^{-1/2}, & qv_0 \ll \Omega_0 \ll qv_n, \quad (1.15a) \\ \Omega_i\left[1+\frac{\varepsilon_0}{\varepsilon_i}\frac{2}{qd}\left(1+\frac{\varkappa_0^2}{q^2}\right)\right]^{-1/2}, & qv_n \ll \Omega_0 \ll qv_0, \quad (1.15b) \\ \sqrt{\frac{2\varepsilon_0\Omega_0^2+\varepsilon_i\Omega_i^2 qd}{2\varepsilon_0+\varepsilon_i qd}}, & qv_n, qv_0 \ll \Omega_0 \quad (1.15c) \end{cases}$$

(there are no oscillations in the frequency range $\omega \sim \Omega_o \ll qv_n, qv_o$).

When the external medium is a dielectric, or an intrinsic semiconductor, in which case all (or almost all) the electrons are strongly bound, $\varepsilon_o(\omega)$ will be in the form

$$\varepsilon_0(\omega) = \varepsilon_0\left[1+\frac{\Omega_0^2 f_0}{\omega_0^2-\omega^2}\right], \tag{1.16}$$

where

ω_o is the natural frequency of the electron junction between levels;

f_o is the corresponding "oscillator force";

(for reasons of simplicity a two-level system without damping is considered).

Now the frequency of the surface oscillations will equal

$$\tilde{\Omega}_q = \sqrt{\omega_0^2+\Omega_q^2 f_0}. \tag{1.17}$$

where Ω_q can be found by using Eq. (1.15a), or (1.15c). The collective excitations with frequency $\tilde{\Omega}_q$ can be called the "surface excitons".

2. The Superconductivity mechanism

We shall, in this section, take up the specific nonphonon mechanism of the superconductivity of quantizing semiconductor (semimetal) films and layered structures of the "sandwich" type, occurring as a result of the interaction of electrons with the acoustic and surface plasma waves[7] already considered in the foregoing, as well as with surface excitons (see [10, 11]). /1599

a. The slit equation. Let us proceed from Gor'kov's [25] equation for "slit" $\Delta(r, r')$ in a heterogeneous superconductor with finite temperatures ($T \neq 0$). Selecting the direction of the X axis perpendicular to the plane of the film, and passing to a Fourier presentation with respect to the longitudinal (homogeneous) Y and Z coordinates for temperature $T \to T_s$, with accuracy to first order terms with respect to Δ, we obtain the following "slit" equation (compare [25, 26])

$$\Delta(\mathrm{q}, \omega_l; x, x') = -e^2T \sum_m{}' \int \frac{d^2k}{(2\pi)^2} \int dx'' \int dx''' \mathfrak{G}_0(-\mathrm{k}, -\omega_m; x''', x) \times$$
$$\times \Delta(\mathrm{k}, \omega_m; x''' x'') \mathfrak{G}_0(\mathrm{k}, \omega_m; x'', x') \mathfrak{D}(\mathrm{q}-\mathrm{k}, \omega_l - \omega_m; x, x'). \quad (2.1)$$

where

$\mathfrak{G}_0$ is Green's temperature function for free (unpaired) conductance electrons, dependent on odd "frequencies" $\omega_m = (2m + 1)\pi T (m = 0, \pm 1, \pm 2, \ldots)$;

$\mathfrak{D}$ is the Green function of a longitudinal electromagnetic field in a three-layered system, obtained by one of the authors of reference [27], which, inside the film (that is, when $0 < x, x' < d$) has the form ($\omega_l = 2l\pi T$);

$$\mathfrak{D}(\mathrm{q}, \omega_l; x, x') = 4\pi\Big\{b(\mathrm{q}, \omega_l; x, x') - \frac{2a(\mathrm{q}, \omega_l; x) \cdot b(\mathrm{q}, \omega_l; 0, x')}{2a(\mathrm{q}, \omega_l; 0) + [q\varepsilon_0(i\omega_l)]^{-1}}\Big\}, \quad (2.2)$$

where

$$a(\mathrm{q}, \omega_l; x) = \frac{1}{d} \sum_\nu \frac{e^{iq_\nu x}}{(q^2 + q_\nu^2)\,\varepsilon(\mathrm{q}, q_\nu, i\omega_l)}, \quad (2.3)$$

$$b(\mathrm{q}, \omega_l; x, x') = \frac{1}{d} \sum_\nu \frac{e^{iq_\nu x} \cdot \cos q_\nu x'}{(q^2 + q_\nu^2)\,\varepsilon(\mathrm{q}, q_\nu, i\omega_l)}. \quad (2.4)$$

---

7. This mechanism is similar to the "plasma" mechanism of superconductivity studied in [24].

We should point out that Eq.(2.2) was obtained without taking the spatial dispersion of the external medium into consideration.

Assuming that the electrons are distributed uniformly through the section of the film[8] such that $\mathcal{G}_0(k,\omega_m;x,x') \equiv \mathcal{G}_0(k,\omega_m;x-x')$, and introducing the Green function $\overline{\mathcal{D}(q,\omega_l;x,x')} \equiv \frac{1}{d}\sum_v \mathcal{D}_v(q,\omega_l)e^{iq_v(x-x')}$, averaged with respect to film thickness (but ignoring the heterogeneity of the field of oscillation, and considering only symmetrical waves with even $q_v = 2v\pi/d$), we obtain the equation for the slit in the longitudinal spectrum of the electrons in the n^{th} subzone as follows in a Fourier presentation with respect to the relative coordinate $(x - x')$ /1600

$$\Delta_n(q,\omega_l) = -\frac{e^2}{d}T\sum_m\sum_{n'=1}^{N}\int\frac{d^2k}{(2\pi)^2}\mathcal{D}_{n-n'}(q-k,\omega_l-\omega_m)\times \\ \times\mathcal{G}^0_{-n'}(-k,-\omega_m)\Delta_{n'}(k,\omega_m)\mathcal{G}^0_{n'}(k,\omega_m), \tag{2.5}$$

where

$$\mathcal{D}_v(q,\omega_l) = \frac{4\pi}{(q^2+q_v^2)\,\varepsilon(q,q_v,i\omega_l)}\times \\ \times\left\{1 - \frac{2}{d(q^2+q_v^2)\,\varepsilon(q,q_v,i\omega_l)\,[2a(q,\omega_l;0)+(q\varepsilon_0(i\omega_l))^{-1}]}\right\}, \tag{2.6}$$

$$\mathcal{G}^0_n(k,\omega_m) = \frac{1}{i\omega_m - \xi_n(k)}; \qquad \xi_n(k) = \frac{\hbar^2}{2m_\parallel}(k^2 - k_n^2). \tag{2.7}$$

As we see, when $d \to \infty$ in the function $\mathcal{D}_v(q,\omega_l)$ is converted into the longitudinal component, $\mathcal{D}_{00}(q,\omega_l)$ of the Green function of an electromagnetic field in a homogeneous medium [22]. In a bounded (heterogeneous) medium $\mathcal{D}_v(q,\omega_l)$ can be described as volume, as well as longitudinal surface oscillations (the latter correspond to the second term in brackets in Eq. (2.6).

When the "Umklapp" processes between subzones can be ignored, and when only terms with $v = 0$ are retained in Eqs. (2.3), (2.4), and (2.6) (see the Eq.(1.3) condition), Eq. (2.5) degenerates into a system of N independent equations for the "slit" in each of the subzones

$$\Delta_n(q,\omega_l) = -\frac{e^2}{d}T\sum_m\int\frac{d^2k}{(2\pi)^2}\mathcal{D}_0(q-k,\omega_l-\omega_m)\frac{\Delta_n(k,\omega_m)}{\omega_m^2+\xi_n^2(k)}, \tag{2.8}$$

---

8. The thickness of the transition region (heterojunction) is considered to be infinitely small compared with d, and with the depth of penetration of the surface waves (approximation of a sharp boundary), and is valid for materials with near workings of the output (see [12]).

where

$$\mathfrak{D}_0(q, \omega_l) = \frac{4\pi d}{q[2\varepsilon_0(i\omega_l) + qd\varepsilon(q, 0, i\omega_l)]}. \tag{2.9}$$

Readily seen is the fact that upon transition to the real frequency, ω, the Green function $\mathfrak{D}_0$ (q, ω_l), has a pole on the natural branches of the oscillations satisfying Eq.(1.4), the dispersion equation; that is, at the frequencies of the acoustic and surface plasma oscillations (see Eqs.(1.10), (1.15), and (1.17)). But since, in accordance with Eq.(2.8), the principle role in the processes of two-dimensional "pairing" of conductance electrons in the n^{th} subzone is that of the transmitted pulses, $\hbar|q-k| \sim \hbar k_n$, the contribution of the virtual acoustic plasmons existing in the boundary regions of wave numbers $q < q_{lim} \lesssim k_n$ and frequencies $\omega < \omega_{lim}$ (see Eqs. (1.10), (1.11)), to the total interelectron interaction is relatively small.[9]

At the same time, the interaction of the electrons with the surface plasmons (excitons) in layered structures of the "sandwich" type can be very much more substantial when conditions are right.

b. Thin films. Let us consider the case of extremely thin films with low permittivity such that $\varepsilon_0 \gg \varepsilon_i \frac{qd}{2}$ $(qd \lesssim 1)$. Then, given the condition that the carriers in the external medium (that is, in the "faces") are free (see Eq. 1.12)), Eq.(2.9) will be in the following form in the range of "frequencies" $\omega_l > \hbar q v_0$ /1601

$$\mathfrak{D}_0(q, \omega_l) = \frac{2\pi d\omega_l^2}{q\varepsilon_0(\omega_l^2 + \hbar^2\Omega_0^2)}. \tag{2.10}$$

Substituting Eq.(2.10) in Eq.(2.8), and integrating with respect to angle θ between vectors q and k in the plane of the film, and aware of

$$\int_0^{2\pi} \frac{d\theta}{|q-k|} = 2\int_0^{\pi} \frac{d\Theta}{\sqrt{q^2 - 2qk\cos\Theta + k^2}} = \frac{4}{q} K\left(\frac{k}{q}\right), \tag{2.11}$$

9. The numerical estimates for slit Δ, obtained in [18] without taking the Coulomb repulsion in the $q > q_{lim}$ region into consideration is greatly inflated. Nevertheless, the quasi-neutral acoustic waves in quantizing films have an indirect (through fluctuations influence on the magnitude of the critical temperature of the superconducting junction (see below).

where

K(p) is a complete elliptic integral of the first type ($p < 1$), and then passing from the variables q, k, to the relative energy $\xi \equiv \xi_n(q), \xi \equiv \xi_n(k)$ we obtain

$$\Delta_n(\xi, \omega_l) = -\alpha_n \sqrt{\frac{\mu_n}{\mu_n + \xi}}\, T \sum_m \frac{(\omega_l - \omega_m)^2}{(\omega_l - \omega_m)^2 + \hbar^2\Omega_0^2} \times$$

$$\times \int_{-\xi}^{\mu_n} Q_n(\xi, \xi') \frac{\Delta_n(\xi', \omega_m)}{\omega_m^2 + \xi'^2} d\xi', \qquad (2.12)$$

where

$$\alpha_n = \frac{e^2}{\varepsilon_0 \hbar v_n}, \quad \mu_n = \frac{\hbar^2 k_n^2}{2m_\parallel}, \quad Q_n(\xi, \xi') = \frac{2}{\pi} K\left(\sqrt{\frac{\mu_n - \xi'}{\mu_n + \xi}}\right). \qquad (2.13)$$

Since $\Delta_n(\xi, \omega_l)$, in concordance with Eq.(2.12), is a slowly changing function of energy ξ, and core $Q_n(\xi, \xi')$ has a peaked maximum when $\xi = \xi' = 0$[10] for slit $\Delta_n(\omega_l) \equiv \Delta_n(0, \omega_l)$ at the boundary of the Fermi n^{th} subzone, we obtain a less complex equation

$$\Delta_n(\omega_l) = -\alpha_n T \sum_m \frac{\Delta_n(\omega_m)(\omega_l - \omega_m)^2}{(\omega_l - \omega_m)^2 + \hbar^2\Omega_0^2} \int_0^{\mu_n} Q_n(0, \xi') \frac{d\xi'}{\omega_m^2 + \xi'^2}. \qquad (2.14)$$

Let us make a qualitative analysis of Eq.(2.14), a linear integral equation, and let us find its approximate solution. As follows from the form in which Eq. (2.14) is written the function $\Delta_n(\omega_l)$ should be an alternating one, and, at the same time, when $l \to \infty$ its asymptotic value is $\Delta_n(l \to \infty \equiv -\Delta_n^\infty < 0$, and when $l = 0$, the slit $\Delta_n(l = 0) \equiv \Delta_n^0$ can, in principle, be positive. In accordance with Eq.(2.14), the characteristic scale of the change in $\Delta_n(\omega_l)$ is the energy $\hbar\Omega_0$, so the "dispersion in the slit, $\Delta_n(\omega_l)$, that is, its dependence on the discrete "frequency" ω_l, can be approximated by the following trial function (compare with [28])

$$\Delta_n(\omega_l) = \begin{cases} \Delta_n^0, & |\omega_l| < \hbar\Omega_0, \\ -\Delta_n^\infty, & |\omega_l| > \hbar\Omega_0. \end{cases} \qquad (2.15)$$

Substituting Eq. (2.15) in Eq. (2.14), and for simplicity, replacing the /1602

10. The elliptic integral K(p), as is known, has the logarithmic characteristic $K(p) \approx \ln \frac{4}{\sqrt{1 - p^2}}$ when $p \to 1$.

elliptic integral K(p) with its minimum value $\pi/2$ when $p = 0$ (that is, when $\xi' = \mu_n$), we obtain the approximate family

$$\Delta_n(l=0) \equiv \Delta_n^0 \approx -\Delta_n^0 \alpha_n T \sum_{|\omega_m|<\hbar\Omega_0} \frac{\omega_m^2}{\omega_m^2+\hbar^2\Omega_0^2} \int_0^{\mu_n} \frac{d\xi}{\omega_m^2+\xi^2} + \tag{2.16}$$
$$+ \Delta_n^\infty \alpha_n T \sum_{|\omega_m|>\hbar\Omega_0} \frac{\omega_m^2}{\omega_m^2+\hbar^2\Omega_0^2} \int_0^{\mu_n} \frac{d\xi}{\omega_m^2+\xi^2},$$

$$\Delta_n(l\to\infty) \equiv -\Delta_n^\infty \approx -\Delta_n^0 \alpha_n T \sum_{|\omega_m|<\hbar\Omega_0} \int_0^{\mu_n} \frac{d\xi}{\omega_m^2+\xi^2} + \tag{2.17}$$
$$+ \Delta_n^\infty \alpha_n T \sum_{|\omega_m|>\hbar\Omega_0} \int_0^{\mu_n} \frac{d\xi}{\omega_m^2+\xi^2}.$$

Ignoring terms of the order $|\omega_m| < \hbar\Omega_o$ in the $\omega_m^2/\hbar^2\Omega_o^2$ region, and replacing the sum of the m integral by ω in the $|\omega_m| > \hbar\Omega_o$ region, we can reduce Eqs.(2.16) and (2.17) to the form ($T = T_c$)

$$\Delta_n^0 = \Delta_n^\infty \frac{\alpha_n}{2} \int_0^{\mu_n} \left[1 - \frac{2}{\pi}\,\mathrm{arctg}\,\frac{\hbar\Omega_0}{\xi}\right] \frac{d\xi}{\xi}, \tag{2.18}$$

$$\Delta_n^\infty = \Delta_n^0 \frac{\alpha_n}{2} \int_0^{\mu_n} \mathrm{th}\,\frac{\xi}{2T_c}\,\frac{d\xi}{\xi} -$$
$$-(\Delta_n^0+\Delta_n^\infty)\frac{\alpha_n}{2} \int_0^{\mu_n} \left[1 - \frac{2}{\pi}\,\mathrm{arctg}\,\frac{\hbar\Omega_0}{\xi}\right] \frac{d\xi}{\xi}. \tag{2.19}$$

The condition for the solvability (self-consistency) of Eqs. (2.18) and (2.19) has the form

$$1 = \frac{\alpha_n^2}{4}\Lambda_n \frac{\ln(2\gamma\mu_n/\pi T_c) - \Lambda_n}{1 + \frac{\alpha_n}{2}\Lambda_n}, \tag{2.20}$$

where

$\ln\gamma = C = 0{,}577\ldots$ is Euler constant,

and the magnitude Λ_n can be found by using the expression

$$\Lambda_n \equiv \sim \int_0^{\mu_n} \left[1 - \frac{2}{\pi} \operatorname{arctg} \frac{\hbar\Omega_0}{\xi}\right] \frac{d\xi}{\xi} \approx \begin{cases} \ln \frac{\mu_n}{\hbar\Omega_0} + \frac{2}{\pi} \frac{\hbar\Omega_0}{\mu_n}, & \mu_n > \hbar\Omega_0, \\ \frac{2}{\pi} \frac{\mu_n}{\hbar\Omega_0}, & \mu_n < \hbar\Omega_0. \end{cases} \tag{2.21}$$

The result is the obtaining of the following exponential formula[11] for the /1603 critical temperature of the superconducting junction for a two-dimensional electron gas in the n^{th} subzone

$$T_c^{(n)} \approx 1{,}14 E^{(n)} \exp\{-1/\varrho_n\}, \tag{2.22}$$

where

$$E^{(n)} = \begin{cases} \hbar\Omega_0, & \mu_n > \hbar\Omega_0, \\ \mu_n, & \hbar\Omega_0 > \mu_n, \end{cases} \tag{2.23}$$

$$\varrho_n = \begin{cases} \dfrac{\alpha_n^2 \left(\ln \frac{\mu_n}{\hbar\Omega_0} + \frac{2}{\pi} \frac{\hbar\Omega_0}{\mu_n}\right)}{4\left[1 + \frac{\alpha_n}{2}\left(\ln \frac{\mu_n}{\hbar\Omega_0} + \frac{2}{\pi} \frac{\hbar\Omega_0}{\mu_n}\right)\left(1 + \frac{\alpha_n}{\pi} \frac{\hbar\Omega_0}{\mu_n}\right)\right]}, & \mu_n > \hbar\Omega_0, \\ \dfrac{\alpha_n}{2} \dfrac{\frac{\alpha_n}{\pi} \frac{\mu_n}{\hbar\Omega_0}}{\left[1 + \frac{\alpha_n}{\pi} \frac{\mu_n}{\hbar\Omega_0}\left(1 + \frac{\alpha_n}{\pi} \frac{\mu_n}{\hbar\Omega_0}\right)\right]}, & \hbar\Omega_0 > \mu_n. \end{cases} \tag{2.24}$$

The dimensionless parameter "electron-plasmon" interaction, ϱ_n, decreases monotonically with increase in $\beta_n = \hbar\Omega_0/\mu_n$, and the preexponential factor E^n increases in proportion to β_n (up to $\beta_n = 1$) in the $\hbar\Omega_0 < \mu_n$ region, and does not depend on β_n in the $\hbar\Omega_0 > \mu_n$ region (that is, when $\beta_n > 1$), so it is easily shown that $T_s^{(n)}$ as a function of β_n reaches a maximum when $\beta_n \approx 1$, that is equal to

$$T_{c\max}^{(n)} \approx 1{,}14\mu_n \cdot \exp\left\{-\frac{2\pi\left[1 + \frac{\alpha_n}{\pi}\left(1 + \frac{\alpha_n}{\pi}\right)\right]}{\alpha_n^2}\right\}. \tag{2.25}$$

But since $\alpha_n = \frac{e^2 m_\parallel}{\epsilon_0 \hbar^2 k_n} \sim \sqrt{1/\mu_n}$, that is, $\mu_n \sim 1/\alpha_n^2$, the critical temperature, in

11. When there are no "Umklapp" processes the individual subzones can be considered as independent "parallel-connected" two-dimensional superconductors with their own critical temperature, $T_c^{(n)}$. The critical temperature for the superconducting junction of a film in this case can be established as the largest of all the $T_c^{(n)}$ values (this usually is $T_c^{(1)}$ for the first subzone).

accordance with Eq. (2.25) is maximum when $\alpha'_n \approx \pi$

$$T^{(n)}_{c\ \max\ \max} \approx 8{,}55\cdot 10^{-3}\,\frac{e^2}{\varepsilon_0^2 a_{\parallel}} \approx 0{,}23\,\frac{m_{\parallel}}{\varepsilon_0^2 m_e'}\ (\mathrm{eV}), \tag{2.26}$$

where

m_e is the free electron mass.

Thus, we obtain the approximate estimate $T^{(n)}_{c\ \max\ \max} \approx 10^2\,^\circ K$, for the parameters $m_{\parallel} \approx m_c$, and $\varepsilon_0 \approx 5$ (at the same time that $\varepsilon_i \lesssim \varepsilon_0$), for example. The condition $\alpha_n \approx \pi$ is, at the same time, determined by the optimum film thickness, d*, for a specified electron concentration, n_0, and the condition $\beta_n \approx 1$, that is, $\hbar\Omega_0 \approx \mu_n$ imposes limitations on the selection of the parameters for the external medium ("faces") in the "sandwich". Specifically, if three subzones in the film are filled (N = 3), $k_1 = \sqrt{\frac{2\pi}{3d^2}(n_0 d^3 + \frac{13}{2}\pi)}$ in the first subzone, and the condition $\alpha_1 \equiv \frac{1}{\varepsilon_0 k_1 a_{\parallel}} = \pi$ links d* with the concentration n_0 as follows /1604

$$n_0\,(d^*)^3 = \frac{3\,(d^*)^2}{2\pi^3\varepsilon_0^2 a_{\parallel}^2} - \frac{13}{2}\pi. \tag{2.27}$$

At the same time, the condition of filling three subzones $\frac{13}{2}\pi < n_0(d^*)^3 < 17\pi$ (see Eq.(1.6)) results in a limitation on the permissible values of d*

$$\pi^2\varepsilon_0 a_{\parallel}\sqrt{26/3} < d^* < \pi^2\varepsilon_0 a_{\parallel}\sqrt{47/3}, \tag{2.28}$$

which, for the above-selected parameters, corresponds to the region of film thicknesses $d \approx 10^{-6}$ cm (at the same time $n_0 \approx (2-5)\cdot 10^{19}\,\mathrm{cm}^{-3}$). Finally, the condition $\hbar\Omega_0 \approx \mu_1 \equiv \hbar^2 k_1^2/2m_{\parallel}$, when $m_0 \approx m_e$, is determined by the optimum concentration of carriers in the "faces" ($N_0^* \approx 10^{18}\,\mathrm{cm}^{-3}$). Thus, in order to obtain the maximum critical temperature for a superconducting junction in layered structures of the "sandwich" type, a whole series of mutually interrelated conditions must be satisfied, and this makes a problem such as this extremely complicated from the experimental standpoint.

Similar consideration can be given to the case of bound electrons in the "faces", the so-called "exciton" mechanism of superconductivity [10, 11]. Now, in place of Eq. (2.14) we use the same assumptions to obtain (see Eq. (1.16))

$$\Delta_n(\omega_l) = -\alpha_n T\sum_m \Delta_n(\omega_m)\,\frac{(\omega_l-\omega_m)^2+\hbar^2\omega_0^2}{(\omega_l-\omega_m)^2+\hbar^2\omega_0^2+\hbar^2\Omega_0^2 f_0}\times$$
$$\times\int_0^{\mu_n} Q_n(0,\xi')\,\frac{d\xi'}{\omega_m^2+\xi'^2}. \tag{2.29}$$

As the calculations show, in this case the magnitude T_c proves to be much lower than in the case of the free carriers in the "faces", because the interaction in the static limit $|\omega_l - \omega_m| = 0$ has the characteristics of repulsion. However, in the case of an inverse population in the levels of bound electrons,[12] when $f_o < 0$, and the condition is $|f_o\Omega_o^2| \gtrsim \omega_o^2$, the sign of the static interaction will reverse (attraction) and relatively high critical temperatures can be obtained[13], at least in principle.

c. "Thick" films. Now let us consider the contrary case relative to thick films with adequately high permittivity, such that $\varepsilon_o \ll \varepsilon_i \frac{qd}{2}$. The Green function of Eq.(2.9) will take the following form in the $\omega_l < \mu_n$ region (see Eqs. (2.13a and (2.14))

$$\mathfrak{D}_0(q, \omega_l) = \frac{4\pi}{\varepsilon_i(q^2 + \varkappa_i^2)} \frac{\omega_l^2}{\omega_l^2 + \hbar^2\Omega_q^2}, \qquad (2.30)$$

where the frequency, Ω_q, is determined by Eq.(1.15a) and $q \sim k_n$.

Substituting Eq.(2.30) in Eq.(2.8), and assuming that by analogy with Eq. (2.15), $\Delta_n(\omega_l) = \Delta_n^o$ when $\omega_l < \hbar\tilde{\Omega}$ and $\Delta_n(\omega_l) = -\Delta_n^\infty$ when $\omega_l > \hbar\tilde{\Omega}$, where $\tilde{\Omega}$ is the mean (effective) frequency of the virtual surface plasmons ($\tilde{\Omega} \ll \Omega_o$), we obtain the following (compare with Eqs.(2.18) and (2.19)) /1605

$$\Delta_n^0 = \Delta_n^\infty \frac{\mathcal{E}_i}{4N} \int_{-\mu_n}^{\mu_n} \frac{d\xi}{\xi\sqrt{(\mathcal{E}_i + \xi)^2 + 4\mathcal{E}_i\mu_n}} \left[1 - \frac{2}{\pi}\operatorname{arctg}\frac{\hbar\tilde{\Omega}}{\xi}\right], \qquad (2.31)$$

$$\Delta_n^\infty \approx \Delta_n^0 \frac{\mathcal{E}_i}{4N} \int_{-\mu_n}^{\mu_n} \frac{d\xi}{\xi\sqrt{(\mathcal{E}_i + \xi)^2 + 4\mathcal{E}_i\mu_n}} \operatorname{th}\frac{\xi}{2T_c} - \qquad (2.32)$$

$$-(\Delta_n^0 + \Delta_n^\infty) \frac{\mathcal{E}_i}{4N} \int_{-\mu_n}^{\mu_n} \frac{d\xi}{\xi\sqrt{(\mathcal{E}_i + \xi)^2 + 4\mathcal{E}_i\mu_n}} \left[1 - \frac{2}{\pi}\operatorname{arctg}\frac{\hbar\tilde{\Omega}}{\xi}\right],$$

---

12. Laser "pumping" can be used to create the non-equilibrium distribution of electrons in the "faces", for example.

13. This question is in need of additional research.

where

$$\mathcal{E}_i = \hbar^2 \varkappa_i^2 / 2m_{\parallel} = 2e^2 N / \varepsilon_i d. \tag{2.33}$$

Assuming that $\mu_n \lesssim \mathcal{E}_i$ (but that $\mu_n \gg \hbar\tilde{\Omega}$), the condition of solvability of Eqs. (2.31) and (2.32) provides us with an equation for finding the critical temperature (compare with Eq. (2.20)

$$1 \approx \frac{\tilde{\varrho}_n^2 \left| \ln \dfrac{\mu_n}{\hbar\tilde{\Omega}} + \dfrac{2}{\pi} \dfrac{\hbar\tilde{\Omega}}{\mu_n} \right|}{1 + \tilde{\varrho}_n \left[\ln \dfrac{\mu_n}{\hbar\tilde{\Omega}} + \dfrac{2}{\pi} \dfrac{\hbar\tilde{\Omega}}{\mu_n} \right]} \ln \frac{1{,}14\hbar\tilde{\Omega}}{T_c}, \tag{2.34}$$

where

$$\tilde{\varrho}_n = \mathcal{E}_i / 2N \sqrt{\mathcal{E}_i (\mathcal{E}_i + 4\mu_n)} . \tag{2.35}$$

As we see, T_c drops very rapidly (exponentially) with increase in the number of filled subzones, N; that is, with increase in film thickness d (or in the electron concentration, n_o). But even if N = 1, the numerical estimates show that the critical temperature cannot exceed units of degrees Kelvin in this case.

In conclusion, it should be pointed out that in solid (macroscopic) quantizing films with characteristic longitudinal dimension $L \gg d \gtrsim n_o^{-1/3}$, and with several filled subzones, the real critical temperature should be markedly lower than T_c, calculated using Eq. (2.26), or Eq. (2.34). This is so because of the logarithmic divergence of the long-wave fluctuations in electron density in infinite two-dimensional systems with collective branches of quasineutral acoustic oscillations when $q \to 0$ (see [10, 11, 29, 30]). But if the film consists of individual microscopic granules (blocks), the size of which is $L \lesssim 10^{-5}$—10^{-4} cm, separated by dielectric barriers reflecting the acoustic plasma waves (such that $q_{min} \sim \pi/L$ is not too small), but "transparent" to Cooper pairs (the thickness of the barrier is far and away less than the coherence length $\xi_o \sim \hbar v_1/\Delta$), the reduction in T_c can be slight[14], and the critical temperature of the superconducting junction in the "sandwich" can be substantial higher than in massive superconductors.

---

14. The "Umklapp" processes between subzones, and the scattering of electrons in the impurities (flaws) can assist in lessening the reduction in T_c.

REFERENCES

/1606

1. Lifshits, I.M., Kaganov, M.I., UFN, 78, 1962, p. 411.

2. Tavger, B.A., Demikhovskiy, V.Ya., UFN, 96, 1968, p. 61.

3. Maksimov, Ye.G., UFN, 88, 1966, p. 587.

4. Shapoval, Ye.A., ZhETF, Pis'ma v redaktsiyu [Letters to the Editor], 5, 1967, p. 57.

5. Strongin, M., et al., Phys. Rev. Lett., 14, 1965, p. 362; Phys. Lett., 17, 1956, p. 224.

6. Matthias, B.T., et al., Phys. Lett., 15, 1965, p. 260.

7. Zavaritskiy, N.V., ZhETF, 57, No. 10, 1969.

8. Strongin, M., et al. Phys. Rev. Lett., 19, 1967, p. 121.

9. Ginzburg, V.L., Kirzhnits, D.A., ZhETF, 46, 1964, p. 397.

10. Ginzburg, V.L., ZhETF, 47, 1964, p. 2318; UFN, 95, 1968, p. 91.

11. Ginzburg, V.L., Kirzhnits, D.A., DAN SSSR, 176, 1967, p. 558.

12. Pashitskiy, E.A., ZhETF, 56, 1969, p. 662; Preprints ITF-68-10, IF-68-8, Kiev, 1968.

13. Ferrel, R.A., Phys. Rev., 111, 1955, p. 1214.

14. Romanov, Yu.A., FTT, 5, 1968, p. 2988.

15. Pashitskiy, E.A., Tomchuk, P.M., ZhETF, Pis'ma v redaktsiyu [Letters to the Editor], 9, 1969, p. 367.

16. Yamaguchi, S., J. Phys. Soc. Japan, 5, 1960, p. 1577.

17. Romanov, Yu.A., Yerukhimov, M.Sh., ZhETF, 55, 1968, p. 1561.

18. Pashitskiy, E.A., Romanov, Yu.A., Preprint IF-69-6, Kiev, 1969.

19. Bardeen, J., Cooper, L.N., Schrieffer, J.R., Phys. Rev., 108, 1957, p. 1175.

20. Bogolyubov, N.N., Tolmachev, V.V., Shirkov, D.V., Novyy metod v teorii sverkhprovodimosti [New Method in the Theory of Superconductivity], Moscow, 1958.

21. Matsubara, T., Progr. Theor. Phys., 14, 1954, p. 352.

22. Abrikosov, A.A., Gor'kov, L.P., Dzyaloshinskiy, N.Ye., Metody kvantovoy teorii polya v statisticheskoy fizike [Quantum Field Theory Methods in Statistical Physics], Moscow, 1962.

23. Romanov, Yu.A., Radiofizika, 7, 1964, p. 828.

24. Pashitskiy, E.A., ZhETF, 55, 1968, p. 2387; Preprints ITF-68-16, IF-68,2, Kiev, 1968.

25. Gor'kov, L.P., ZhETF, 36, 1959, p. 1918.

26. Geylikman, B.T., Kresin, V.Z., DAN SSSR, 182, 1968, p. 1040.

27. Romanov, Yu.A., Candidate Dissertation, Gor'kiy State University, 1966; ZhETF, 47, 1964, p. 2119.

28. McMillan, W.L., Phys. Rev., 167, 1968, p. 331.

29. Bogoliubov, N.N., Physica, 26, 1960, p. 51.

30. Hohenberg, P.C., Phys. Rev., 158, 1967, p. 383.

Translated for the National Aeronautics and Space Administration under contract NASw-2038 by Translation Consultants, Ltd., Arlington, Virginia.